Samples **Leslie matrix SOLUTIONS**

1. The initial population vector P_0 and the Leslie matrix L are:

$$P_0 = \begin{pmatrix} 20 \\ 2 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 1 & 7 & 4 & 1 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{pmatrix}.$$

Then to find the population at time step $t+1$ we calculate $P_{t+1} = L \times P_t$, as follows:
At time $t = 1$ the population P_1 is given by:

$$\begin{pmatrix} 1 & 7 & 4 & 1 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{pmatrix} \times \begin{pmatrix} 20 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \times 20 + 7 \times 2 + 4 \times 1 + 1 \times 1 \\ 0.4 \times 20 + 0 \times 2 + 0 \times 1 + 0 \times 1 \\ 0 \times 20 + 0.8 \times 2 + 0 \times 1 + 0 \times 1 \\ 0 \times 20 + 0 \times 2 + 0.4 \times 1 + 0 \times 1 \end{pmatrix} = \begin{pmatrix} 39 \\ 8 \\ 1.6 \\ 0.4 \end{pmatrix}$$

At time $t = 2$ the population P_2 is given by:

$$\begin{pmatrix} 1 & 7 & 4 & 1 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{pmatrix} \times \begin{pmatrix} 39 \\ 8 \\ 1.6 \\ 0.4 \end{pmatrix} = \begin{pmatrix} 1 \times 39 + 7 \times 8 + 4 \times 1.6 + 1 \times 0.4 \\ 0.4 \times 39 + 0 \times 8 + 0 \times 1.6 + 0 \times 0.4 \\ 0 \times 39 + 0.8 \times 8 + 0 \times 1.6 + 0 \times 0.4 \\ 0 \times 39 + 0 \times 8 + 0.4 \times 1.6 + 0 \times 0.4 \end{pmatrix} = \begin{pmatrix} 101.8 \\ 15.6 \\ 6.4 \\ 0.6 \end{pmatrix}$$

2. The initial population vector P_0 and the Leslie matrix L are:

$$P_0 = \begin{pmatrix} 12 \\ 2 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 3 & 9 \\ 0.5 & 0 \end{pmatrix}.$$

Then to find the population at time step $t+1$ we calculate $P_{t+1} = L \times P_t$, as follows:
At time $t = 1$ the population P_1 is given by:

$$\begin{pmatrix} 3 & 9 \\ 0.5 & 0 \end{pmatrix} \times \begin{pmatrix} 12 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \times 12 + 9 \times 2 \\ 0.5 \times 12 + 0 \times 2 \end{pmatrix} = \begin{pmatrix} 54 \\ 6 \end{pmatrix}$$

At time $t = 2$ the population P_2 is given by:

$$\begin{pmatrix} 3 & 9 \\ 0.5 & 0 \end{pmatrix} \times \begin{pmatrix} 54 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \times 54 + 9 \times 6 \\ 0.5 \times 54 + 0 \times 6 \end{pmatrix} = \begin{pmatrix} 216 \\ 27 \end{pmatrix}$$

3. The initial population vector P_0 and the Leslie matrix L are:

$$P_0 = \begin{pmatrix} 9 \\ 2 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 1 & 3 & 3 & 0 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{pmatrix}.$$

Then to find the population at time step $t+1$ we calculate $P_{t+1} = L \times P_t$, as follows:
At time $t = 1$ the population P_1 is given by:

$$\begin{pmatrix} 1 & 3 & 3 & 0 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{pmatrix} \times \begin{pmatrix} 9 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \times 9 + 3 \times 2 + 3 \times 1 + 0 \times 1 \\ 0.3 \times 9 + 0 \times 2 + 0 \times 1 + 0 \times 1 \\ 0 \times 9 + 0.9 \times 2 + 0 \times 1 + 0 \times 1 \\ 0 \times 9 + 0 \times 2 + 0.3 \times 1 + 0 \times 1 \end{pmatrix} = \begin{pmatrix} 18 \\ 2.7 \\ 1.8 \\ 0.3 \end{pmatrix}$$

At time $t = 2$ the population P_2 is given by:

$$\begin{pmatrix} 1 & 3 & 3 & 0 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{pmatrix} \times \begin{pmatrix} 18 \\ 2.7 \\ 1.8 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 1 \times 18 + 3 \times 2.7 + 3 \times 1.8 + 0 \times 0.3 \\ 0.3 \times 18 + 0 \times 2.7 + 0 \times 1.8 + 0 \times 0.3 \\ 0 \times 18 + 0.9 \times 2.7 + 0 \times 1.8 + 0 \times 0.3 \\ 0 \times 18 + 0 \times 2.7 + 0.3 \times 1.8 + 0 \times 0.3 \end{pmatrix} = \begin{pmatrix} 31.5 \\ 5.4 \\ 2.4 \\ 0.5 \end{pmatrix}$$