Samples **Floating SOLUTIONS**

1. **(1)** Let us denote the mass and volume of the block as b and V respectively, the mass of Ben as m, the density of water as ρ_w and the density of the block as ρ_b. The mass of water w displaced by the block is then:

$$\begin{aligned} w &= \rho_w \times V \\ &= \rho_w \times \frac{b}{\rho_b} \\ &= \frac{\rho_w}{\rho_b} \times b \end{aligned}$$

The minimum amount of foam needed will be that which keeps the system in equilibrium (i.e. any more and the block will rise, any less and it will sink). Thus, the mass of water displaced must equal Ben's mass *as well as the mass of the block!*.

$$\begin{aligned} & w &= m + b \\ \Longrightarrow & \frac{\rho_w}{\rho_b} \times b &= m + b \\ \Longrightarrow & \frac{\rho_w}{\rho_b} \times b - b &= m \\ \Longrightarrow & \left(\frac{\rho_w}{\rho_b} - 1\right) \times b &= m \\ \Longrightarrow & b &= \frac{m}{\frac{\rho_w}{\rho_b} - 1} \end{aligned}$$

Substituting in the values and rounding off to two decimal places, we have:

$$\begin{aligned} b &= \frac{83}{\frac{1000.0}{12} - 1} \\ &= 1.01\text{kg} \end{aligned}$$

Thus, Ben will need at least 1.01kg of foam to support his weight.

(2) Using the same symbols as above, let us further denote the percentage of the block that is *underwater* by p. That is:

$$p = 100\% - 45\% = 55\% = 0.55$$

The mass of water that is displaced is now thus dependent upon how much of the block is underwater, V', where:

$$V' = p \times V$$

Thus, the mass of water w' displaced with this reduced volume will be:

$$\begin{aligned} w' &= \rho_w \times V' \\ &= \rho_w \times (p \times V) \\ &= p\rho_w \times \frac{b}{\rho_b} \\ &= \frac{p\rho_w}{\rho_b} \times b \end{aligned}$$

Again, we want the block to be in equilibrium, so:

$$\begin{aligned} w' &= m + b \\ \Longrightarrow \quad \frac{p\rho_w}{\rho_b} \times b &= m + b \\ \Longrightarrow \quad \frac{p\rho_w}{\rho_b} \times b - b &= m \\ \Longrightarrow \quad \left(\frac{p\rho_w}{\rho_b} - 1\right) \times b &= m \\ \Longrightarrow \quad b &= \frac{m}{\frac{p\rho_w}{\rho_b} - 1} \end{aligned}$$

Substituting in the values and rounding off to two decimal places, we have:

$$\begin{aligned} b &= \frac{83}{\frac{0.55 \times 1000.0}{12} - 1} \\ &= 1.85\text{kg} \end{aligned}$$

Thus, Ben will need at least 1.85kg of foam to support his weight if he wants to ensure that at least 45% of it remains above water.

2. **(1)** Let us denote the mass and volume of the block as b and V respectively, the mass of Ben as m, the density of water as ρ_w and the density of the block as ρ_b. The mass of water w displaced by the block is then:

$$\begin{aligned} w &= \rho_w \times V \\ &= \rho_w \times \frac{b}{\rho_b} \\ &= \frac{\rho_w}{\rho_b} \times b \end{aligned}$$

The minimum amount of foam needed will be that which keeps the system in equilibrium (i.e. any more and the block will rise, any less and it will sink). Thus, the mass of water displaced must equal Ben's mass *as well as the mass of the block!*.

$$\begin{aligned} w &= m + b \\ \Longrightarrow \quad \frac{\rho_w}{\rho_b} \times b &= m + b \\ \Longrightarrow \quad \frac{\rho_w}{\rho_b} \times b - b &= m \\ \Longrightarrow \quad \left(\frac{\rho_w}{\rho_b} - 1\right) \times b &= m \\ \Longrightarrow \quad b &= \frac{m}{\frac{\rho_w}{\rho_b} - 1} \end{aligned}$$

Substituting in the values and rounding off to two decimal places, we have:

$$\begin{aligned} b &= \frac{106}{\frac{1000.0}{19} - 1} \\ &= 2.05\text{kg} \end{aligned}$$

Thus, Ben will need at least 2.05kg of foam to support his weight.

(**2**) Using the same symbols as above, let us further denote the percentage of the block that is *underwater* by p. That is:

$$p = 100\% - 57\% = 43\% = 0.43$$

The mass of water that is displaced is now thus dependent upon how much of the block is underwater, V', where:

$$V' = p \times V$$

Thus, the mass of water w' displaced with this reduced volume will be:

$$\begin{aligned} w' &= \rho_w \times V' \\ &= \rho_w \times (p \times V) \\ &= p\rho_w \times \frac{b}{\rho_b} \\ &= \frac{p\rho_w}{\rho_b} \times b \end{aligned}$$

Again, we want the block to be in equilibrium, so:

$$\begin{aligned} && w' &= m + b \\ \implies && \frac{p\rho_w}{\rho_b} \times b &= m + b \\ \implies && \frac{p\rho_w}{\rho_b} \times b - b &= m \\ \implies && \left(\frac{p\rho_w}{\rho_b} - 1\right) \times b &= m \\ \implies && b &= \frac{m}{\frac{p\rho_w}{\rho_b} - 1} \end{aligned}$$

Substituting in the values and rounding off to two decimal places, we have:

$$\begin{aligned} b &= \frac{106}{\frac{0.43 \times 1000.0}{19} - 1} \\ &= 4.90\text{kg} \end{aligned}$$

Thus, Ben will need at least 4.90kg of foam to support his weight if he wants to ensure that at least 57% of it remains above water.

3. (**1**) Let us denote the mass and volume of the block as b and V respectively, the mass of Ben as m, the density of water as ρ_w and the density of the block as ρ_b. The mass of water w displaced by the block is then:

$$\begin{aligned} w &= \rho_w \times V \\ &= \rho_w \times \frac{b}{\rho_b} \\ &= \frac{\rho_w}{\rho_b} \times b \end{aligned}$$

The minimum amount of foam needed will be that which keeps the system in equilibrium (i.e. any more and the block will rise, any less and it will sink). Thus, the mass of water displaced must equal Ben's mass *as well as the mass of the block!*.

$$
\begin{aligned}
w &= m + b \\
\implies \quad \frac{\rho_w}{\rho_b} \times b &= m + b \\
\implies \quad \frac{\rho_w}{\rho_b} \times b - b &= m \\
\implies \quad \left(\frac{\rho_w}{\rho_b} - 1\right) \times b &= m \\
\implies \quad b &= \frac{m}{\frac{\rho_w}{\rho_b} - 1}
\end{aligned}
$$

Substituting in the values and rounding off to two decimal places, we have:

$$
\begin{aligned}
b &= \frac{99}{\frac{1000.0}{19} - 1} \\
&= 1.92\text{kg}
\end{aligned}
$$

Thus, Ben will need at least 1.92kg of foam to support his weight.

(2) Using the same symbols as above, let us further denote the percentage of the block that is *underwater* by p. That is:

$$p = 100\% - 32\% = 68\% = 0.68$$

The mass of water that is displaced is now thus dependent upon how much of the block is underwater, V', where:

$$V' = p \times V$$

Thus, the mass of water w' displaced with this reduced volume will be:

$$
\begin{aligned}
w' &= \rho_w \times V' \\
&= \rho_w \times (p \times V) \\
&= p\rho_w \times \frac{b}{\rho_b} \\
&= \frac{p\rho_w}{\rho_b} \times b
\end{aligned}
$$

Again, we want the block to be in equilibrium, so:

$$
\begin{aligned}
w' &= m + b \\
\implies \quad \frac{p\rho_w}{\rho_b} \times b &= m + b \\
\implies \quad \frac{p\rho_w}{\rho_b} \times b - b &= m \\
\implies \quad \left(\frac{p\rho_w}{\rho_b} - 1\right) \times b &= m \\
\implies \quad b &= \frac{m}{\frac{p\rho_w}{\rho_b} - 1}
\end{aligned}
$$

Substituting in the values and rounding off to two decimal places, we have:

$$
\begin{aligned}
b &= \frac{99}{\frac{0.68 \times 1000.0}{19} - 1} \\
&= 2.85\text{kg}
\end{aligned}
$$

Thus, Ben will need at least 2.85kg of foam to support his weight if he wants to ensure that at least 32% of it remains above water.